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ABSORPTION OF FIXED SCALARS

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Abstract

We calculate the absorption rates of fixed scalars by near extremal five dimensional black holes carrying general one-brane and five-brane charges by semi-classical and D-brane methods. We find that the absorption cross-sections do not in general agree for either fixed scalar and we discuss possible explanations of the discrepancy.

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I. INTRODUCTION

In the past year there has been a great deal of interest in a certain class of five-dimensional black holes with three gauge charges whose properties may be reproduced by an effective string model based on intersecting D-branes. Initially it was shown that such a model could reproduce the Bekenstein-Hawking entropy of these black holes [1], [2], [3], [4], [5] and more recently detailed comparisons of the emission rates have been made [6], [7], [8], [9], [10], [11], [12], [13], [14] [15], [16]. Such calculations are straightforward both semi-classically and in the string model for minimally coupled scalar fields.

However for minimally coupled scalars the absorption rate is not a sensitive function of the moduli and energy; indeed for low energy neutral scalar emission the absorption cross-section depends only on the horizon area [17]. A better test of the agreement between semi-classical and string model calculations is provided by fixed scalars [18] which are coupled to the gauge fields and whose absorption cross-sections behave very differently to those of minimally coupled scalars. In [19], [20], the string calculation for a particular fixed scalar, which is related to the volume of the internal torus around which five branes wrap, was found to agree with the semi-classical calculation in the case that the one and five brane charges are the same. A particularly interesting feature of this calculation is that the string cross-section involves the tension of the effective string and thus one might expect a calculation of the semi-classical cross-section for general charges to confirm that this tension is independent of the one-brane charge.

Although the string calculation was possible for all one and five brane charges within the dilute gas region, the semi-classical calculations were found to be technically difficult, owing to a coupling between fixed scalar and gravitational perturbations when the charges were not the same. In this paper we discuss the solution of the semi-classical field equations for the fixed scalars when the charges take general values. We calculate the absorption cross-sections for the two fixed scalars related to the volume of the internal torus and the scale of the effective string direction respectively. We then approximate the cross-section for the latter derived from the effective string action, and compare the results.

We find that for general values of the charges the cross-sections for the fixed scalars calculated in the two different regimes do not agree. There is a mixing between the two fixed scalars at the semi-classical level when the charges are not equal, whereas there does not appear to be any such mixing from the effective string point of view. In the limit of low energy absorption from a very near extremal state, the string cross-sections behave as T_L^5, where T_L is the temperature of the left movers, whereas the semi-classical cross-sections behave as $\omega^2 T_L^3$, where ω is the frequency. This discrepancy derives from the presence of chiral operators of dimensions $(3,1)$ and $(1,3)$ as well as an operator of dimension $(2,2)$ in the string interaction for one fixed scalar, whilst no such operators contribute to the interactions of the other.

The paper is organised as follows. In section II we discuss the semi-classical calculation of the absorption cross-sections for the two fixed scalars, and in section III we calculate the functional dependence of the absorption cross-sections implied by the string theory effective action. We give our conclusions in section IV.

II. SEMI-CLASSICAL CALCULATION OF GREYBODY FACTORS

As usual we consider a class of five-dimensional black hole representing the bound state of n_1 RR strings and n_5 RR 5-branes compactified on a 5-torus first discussed in [1], [2]. These black holes can be regarded as a solution to a truncation of type IIB superstring effective action compactified on a 5-torus

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}[R - (\partial\phi)^2 - \frac{4}{3}(\partial\lambda)^2 - 4(\partial\nu)^2 \\ -\frac{1}{4}e^{\frac{8\lambda}{3}}F^{(K)2} - \frac{1}{4}e^{-\frac{4\lambda}{3}+4\nu}F^2 - \frac{1}{4}e^{-\frac{4\lambda}{3}-4\nu}H^2] \tag{1}$$

where $F^{(K)}_{\mu\nu}$ is the Kaluza-Klein vector field strength corresponding to the string direction and $F_{\mu\nu}$, $H_{\mu\nu}$ are the "electric" and "magnetic" components of the field strength of the RR two form. As discussed in [19], the dilaton ϕ is a decoupled scalar, whilst the scalars λ and ν interact with the gauge fields and are examples of fixed scalars [18]. The scalar λ is related to the scale of the Kaluza-Klein circle whilst ν is related to the scale of the internal torus.

Following [19], we choose the five dimensional metric as

$$ds_5^2 = -e^{2a(t,r)}dt^2 + e^{2b(t,r)}dr^2 + e^{2c(t,r)}d\Omega_3^2, \tag{2}$$

where we assume that the metric functions are angular independent since we are interested in low energy scattering for which only $l = 0$ components are significant. Now the graviton equations of motion take the forms

$$R_{\mu\nu} = \frac{4}{3}\partial_\mu\lambda\partial_\nu\lambda + 4\partial_\mu\nu\partial_\nu\nu + T^M_{\mu\nu}, \tag{3}$$

where the trace adjusted energy tensor for the gauge fields is

$$T^M_{\mu\nu} = e^{\frac{8\lambda}{3}}(\frac{1}{2}F^{(K)}_{\mu\lambda}F^{(K)\lambda}_{\nu} - \frac{1}{12}F^{(K)2}g_{\mu\nu}) + e^{-\frac{4\lambda}{3}+4\nu}(\frac{1}{2}F_{\mu\lambda}F^{\lambda}_{\nu} \\ -\frac{1}{12}F^2 g_{\mu\nu}) + \frac{1}{4}e^{-\frac{4\lambda}{3}-4\nu}(\frac{1}{2}H_{\mu\lambda}H^{\lambda}_{\nu} - \frac{1}{12}H^2 g_{\mu\nu}), \tag{4}$$

and we have omitted dilaton components, since we can choose the background value of the dilaton to be constant and can set all variations to vanish. Solving the equations of motion for the gauge fields we find that

$$F^{(K)}_{tr} = 2Q_K e^{a+b-3c-\frac{8}{3}\lambda} \tag{5}$$

$$F_{tr} = 2Qe^{a+b-3c+\frac{4}{3}\lambda-4\nu} \tag{6}$$

$$H_{tr} = 2Pe^{a+b-3c+\frac{4}{3}\lambda+4\nu} \tag{7}$$

where Q_K, Q and P are the gauge charges. We can then eliminate these fields from the action, as in [19], replacing them with a potential dependent on the fixed scalars and the metric function c only. Thus the equation for the fixed scalar ν takes the form

$$\partial_t(e^{-a+b+3c}\dot{\nu}) - \partial_r(e^{a-b+3c}\nu') = e^{a+b-3c}[-P^2 e^{\frac{4\lambda}{3}+4\nu} + Q^2 e^{\frac{4\lambda}{3}-4\nu}], \tag{8}$$

whilst the equation for the fixed scalar λ is

$$\partial_t(e^{-a+b+3c}\dot{\lambda}) - \partial_r(e^{a-b+3c}\lambda') = e^{a+b-3c}[2Q_K^2 e^{\frac{8\lambda}{3}} - P^2 e^{\frac{4\lambda}{3}+4\nu} - Q^2 e^{\frac{4\lambda}{3}-4\nu}]. \quad (9)$$

We are interested in linearising these equations of motion about the background metric

$$e^{2a_0} = h\mathcal{H}^{-2/3}, e^{2b_0} = h^{-1}\mathcal{H}^{1/3}, e^{2c_0} = r^2\mathcal{H}^{1/3}, \quad (10)$$

in which the fixed scalars are given by

$$e^{2\lambda_0} = H_{r_K^2}(H_{r_1^2}H_{r_5^2})^{-1/2}, e^{4\nu_0} = H_{r_1^2}H_{r_5^2}^{-1}, \quad (11)$$

and the metric functions are defined as

$$h = (1 - \frac{r_0^2}{r^2}), H_{r_i^2} = 1 + \frac{r_i^2}{r^2}, \quad (12)$$
$$r_i^2 = \sqrt{Q_i^2 + \frac{1}{4}r_0^4} - \frac{1}{2}r_0^2, \mathcal{H} = H_{r_1^2}H_{r_5^2}H_{r_K^2},$$

where we have introduced the characteristic radii r_i and r_0^2 is an extremality parameter. In the extremal limit, $r_1^2 = Q$, $r_5^2 = P$ and $r_K^2 = Q_K$. We shall be interested in the dilute gas limit for which $r_K = r_0 \sinh\sigma$, and the effective temperatures of the left and right moving modes on the effective string are related to the radial parameters as

$$T_L = \frac{1}{2\pi}(\frac{r_0}{r_1 r_5})e^{\sigma}, \quad T_R = \frac{1}{2\pi}(\frac{r_0}{r_1 r_5})e^{-\sigma}. \quad (13)$$

To obtain the linearised equations of motion, we let the scalar functions take the form $f = f_0 + \delta f$. We will use the residual gauge freedom to fix the metric on the sphere, i.e. $\delta c = 0$. Since the background is static, we can replace all coefficients of time derivatives in the equations of motion with their background values. For the radial-time graviton equation this produces the simplified linearised equation

$$R_{tr} = \frac{4}{3}\lambda_0'\delta\dot{\lambda} + 4\nu_0'\delta\dot{\nu}, \quad (14)$$

and from the form of the metric we can calculate the relevant component of the Ricci tensor as $3c_0'\delta\dot{b}$ so that

$$\delta b = \frac{1}{3c_0'}(\frac{4}{3}\lambda_0'\delta\lambda + 4\nu_0'\delta\nu) \quad (15)$$

where we have integrated with respect to the time coordinate.

It is then convenient to use the angular graviton equations of motion; the angular components of the Ricci tensor depend only on the zeroth and first derivatives of the perturbations, whilst the time and radial components depend also on second derivatives. That is,

$$\delta R_{\theta\theta} = g_{\theta\theta}e^{-2b_0}\{c_0'(\delta b' - \delta a') + 2(c_0'(a_0' - b_0' + 3c_0') + c_0'')\delta b\}, \quad (16)$$

where θ is a coordinate on the sphere. From the graviton equation of motion, however, we find that

$$\delta R_{\theta\theta} = \frac{1}{12} g_{\theta\theta} \delta [e^{\frac{8}{3}\lambda} F^{(K)2} + e^{-\frac{4}{3}\lambda + 4\nu} F^2 + e^{-\frac{4}{3}\lambda - 4\nu} H^2]. \tag{17}$$

We hence find a relation between the first derivatives of the gravitational perturbations

$$\begin{aligned}(\delta a' - \delta b') - f(r)\delta b = \frac{2\mathcal{H}^{-2/3}}{3r^6 h c_0'} [& \frac{8Q_K^2}{3} e^{-\frac{8}{3}\lambda_0} \delta\lambda - Q^2 e^{\frac{4}{3}\lambda_0 - 4\nu_0} (\frac{4}{3}\delta\lambda - 4\delta\nu) \\ & - P^2 e^{\frac{4}{3}\lambda_0 + 4\nu_0} (\frac{4}{3}\delta\lambda + 4\delta\nu)],\end{aligned} \tag{18}$$

where the function $f(r)$ is given by

$$f(r) = 2(\frac{c_0''}{c_0'} + \frac{3}{r} + \frac{h'}{h}) \tag{19}$$

Given the two equations (15) and (18) for the gravitational perturbations, we can substitute into the linearised equations for the fixed scalars

$$e^{-a_0+b_0+3c_0} \delta\ddot{\nu} - \delta(\partial_r(e^{a-b+3c_0}\delta\nu')) = \delta(e^{a+b-3c_0}[-P^2 e^{\frac{4\lambda}{3}+4\nu} + Q^2 e^{\frac{4\lambda}{3}-4\nu}]), \tag{20}$$

$$e^{-a_0+b_0+3c_0} \delta\ddot{\lambda} - \delta(\partial_r(e^{a-b+3c_0}\lambda')) = \delta\{e^{a+b-3c_0}[2Q_K^2 e^{\frac{8\lambda}{3}} - P^2 e^{\frac{4\lambda}{3}+4\nu} - Q^2 e^{\frac{4\lambda}{3}-4\nu}]\}, \tag{21}$$

and decouple the equations for the fixed scalars from those for the gravitational perturbations. For modes of frequency ω such that $\delta f = \tilde{f} e^{i\omega t}$, we find the following coupled equations for the fixed scalars,

$$[(hr^3\partial_r)^2 + \omega^2 r^6 \mathcal{H} + F_\lambda]\tilde{\lambda} + 3F(r)\tilde{\nu} = 0 \tag{22}$$

$$[(hr^3\partial_r)^2 + \omega^2 r^6 \mathcal{H} + F_\nu]\tilde{\nu} + F(r)\tilde{\lambda} = 0, \tag{23}$$

where the functions are given by

$$\begin{aligned}F_\lambda = -\frac{8hr^4}{(r_1^2 r_5^2 + r_1^2 r_K^2 + r_5^2 r_K^2 + 2(r_1^2 + r_5^2 + r_K^2) r^2 + 3r^4)^2} \{ & r_1^4 r_5^4 + r_1^4 r_K^4 + r_5^4 r_K^4 \\ & + 2r_1^2 r_5^2 r_K^2 (r_1^2 + r_5^2 + r_K^2) + ((r_1^2 r_5^2 + 4r_K^4) + r_K^2(r_1^2 + r_5^2) + r_K^2(r_1^4 + r_5^4) \\ & + 6r_1^2 r_5^2 r_K^2) r^2 + (r_1^4 + r_5^4 - r_1^2 r_5^2 + 4r_K^4 + 2r_K^2 r_1^2 + 2r_K^2 r_5^2) r^4 \},\end{aligned} \tag{24}$$

and

$$\begin{aligned}F_\nu = -\frac{8hr^4}{(r_1^2 r_5^2 + r_1^2 r_K^2 + r_5^2 r_K^2 + 2(r_1^2 + r_5^2 + r_K^2) r^2 + 3r^4)^2} \{ & r_1^4 r_5^4 + r_1^4 r_K^4 + r_5^4 r_K^4 \\ & + 2r_1^2 r_5^2 r_K^2 (r_1^2 + r_5^2 + r_K^2) + 3(r_1^2 r_5^2 (r_1^2 + r_5^2) + (r_1^4 + r_5^4) r_K^2 + 2r_1^2 r_5^2 r_K^2) r^2 \\ & + 3(r_1^4 + r_5^4 + r_1^2 r_5^2) r^4 \},\end{aligned} \tag{25}$$

and

$$F(r) = 8hr^6 \frac{(r_1^2 - r_5^2)[r_1^2 r_5^2 + r_1^2 r_K^2 + r_5^2 r_K^2 + (r_1^2 + r_5^2 + r_K^2) r^2]}{(r_1^2 r_5^2 + r_1^2 r_K^2 + r_5^2 r_K^2 + 2(r_1^2 + r_5^2 + r_K^2) r^2 + 3r^4)^2}. \tag{26}$$

For notational simplicity, we have assumed that the black hole is very near extremal, so that $r_0^2 \ll r_1^2, r_5^2$, and we accordingly retain only leading order terms. Note that when the one and five brane charges are equal the equation for the fixed scalar $\tilde{\nu}$ reduces to

$$[(hr^3\partial_r)^2 + \omega^2 r^6 \mathcal{H} - \frac{8hr^4R^4}{(r^2+R^2)^2}(1+\frac{r_0^2}{R^2})]\tilde{\nu} = 0, \tag{27}$$

where we have set $r_1 = r_5 \equiv R$; this is indeed the equation obtained in [19]. Note that we have restored the full dependence on the non extremality parameter so that the equation is exact. The corresponding equation for the fixed scalar $\tilde{\lambda}$ reduces to

$$[(hr^3\partial_r)^2 + \omega^2 r^6 \mathcal{H} - \frac{8hr^4(R^2+2r_K^2)^2}{(3r^2+(R^2+2r_K^2))^2}(1+\frac{r_0^2}{(R^2+2r_K^2)})]\tilde{\lambda} = 0, \tag{28}$$

which in the dilute gas region $r_K \ll R$ differs from the equation for the other fixed scalar by only a factor in the effective potential term.

Now the equations for the fixed scalars are in general coupled, although it is apparent from the form of $F(r)$ that the equations decouple when the one and five brane charges are equal as was found in [19]. We can however find a linear transformation of the fields which decouples the equations; introducing two scalar fields ϕ_a, ϕ_b the required transformation is

$$\begin{aligned} \tilde{\lambda} &= \sqrt{3}[\phi_a \cos\psi - \phi_b \sin\psi], \\ \tilde{\nu} &= \phi_a \sin\psi + \phi_b \cos\psi, \end{aligned} \tag{29}$$

where the mixing angle ψ is defined by the quadratic equation

$$\tan^2\psi - \frac{2}{\sqrt{3}}(\frac{r_1^2+r_5^2-2r_K^2}{r_1^2-r_5^2})\tan\psi - 1 = 0, \tag{30}$$

and hence we find that

$$\sin^2\psi = \frac{1}{2} \pm \frac{1}{4}\frac{r_1^2+r_5^2-2r_K^2}{\sqrt{r_1^4+r_5^4+r_K^4-r_1^2r_5^2-r_1^2r_K^2-r_5^2r_K^2}}. \tag{31}$$

The transformed fields satisfy the equations

$$[(hr^3\partial_r)^2 + \omega^2 r^6 \mathcal{H} - 8\frac{hr^4r_{a,b}^4}{(r^2+r_{a,b}^2)^2}(1+\frac{r_0^2}{r_{a,b}^2})]\phi_{a,b} = 0, \tag{32}$$

where the effective radii $r_{a,b}$ are defined as

$$r_a^2 = \frac{1}{3}[r_1^2+r_5^2+r_K^2+\sqrt{r_1^4+r_5^4+r_K^4-r_1^2r_5^2-r_1^2r_K^2-r_5^2r_K^2}], \tag{33}$$

$$r_b^2 = \frac{1}{3}[r_1^2+r_5^2+r_K^2-\sqrt{r_1^4+r_5^4+r_K^4-r_1^2r_5^2-r_1^2r_K^2-r_5^2r_K^2}]. \tag{34}$$

Now when the one and five brane charges are equal, we choose $r_a \equiv R$, $\tilde{\nu} \equiv \phi_a$ and $\tilde{\lambda} \equiv \sqrt{3}\phi_b$. This implies $\sin\psi = 1$ and fixes the sign as being positive in (31).

The equations for the transformed fields (32) have the same form as the equation for the fixed scalar ν, which was solved first in [19] under certain conditions (extremality and low energy very near extremality) and later in [20] under more general conditions. As usual there does not appear to be an analytic solution, and we must patch together a solution between the near region (region I, $r \ll r_i$), the intermediate region (region II, $r_0 \ll r \ll \omega^{-1}$) and the far region (region III, $r \gg r_i$).

Following [19] and [20], we may write down the dominant terms and the approximate solutions in the three regions as

$$\begin{array}{ll} \mathrm{I.}[(hr^3\partial_r)^2 + r_1^2 r_5^2 (r^2 + r_K^2)\omega^2 - 8r^4 h]\phi^I_{a,b} = 0 & \phi^I_{a,b} = E\frac{r^2}{r_0^2} + G; \\ \mathrm{II.}[(r^3\partial_r)^2 - 8\frac{r^4_{a,b}}{(1+\frac{r^2_{a,b}}{r^2})^2}]\phi^{II}_{a,b} = 0 & \phi^{II}_{a,b} = \frac{C_{a,b}}{(1+\frac{r^2_{a,b}}{r^2})} + D_{a,b}(1+\frac{r^2_{a,b}}{r^2})^2; \\ \mathrm{III.}[(r^3\partial_r)^2 + r^6\omega^2]\phi^{III}_{a,b} = 0 & \phi^{III}_{a,b} = \alpha_{a,b}\frac{J_1(\omega r)}{\omega r} + \beta_{a,b}\frac{N_1(\omega r)}{\omega r}, \end{array} \tag{35}$$

where $C_{a,b}$, $D_{a,b}$, $\alpha_{a,b}$, $\beta_{a,b}$ are constants. The full solution in the inner region is obtained in terms of hypergeometric functions [20], and we give only the limiting form for $r \to r_0$. The quantity E is fixed by the requirement that the solutions are purely ingoing at the horizon to be

$$E = \frac{2\Gamma(1-ia-ib)}{\Gamma(2-ia)\Gamma(2-ib)}, \tag{36}$$

The quantity G is similarly fixed, but we will not need it here. The constants a and b are related to the effective left and right moving temperatures as

$$a = \frac{\omega}{4\pi T_L}, \qquad b = \frac{\omega}{4\pi T_R}. \tag{37}$$

Matching between the three regions we need only retain the constants

$$\alpha_{a,b} = 2A_{a,b} = 2E\frac{r^2_{a,b}}{r_0^2}, \tag{38}$$

provided that we calculate the absorption cross-sections by the ratio of fluxes method [12]. The absorption probability for each scalar is given by the ratio of the incoming fluxes at the horizon and at infinity where the flux of a scalar field f is given by

$$F = \frac{1}{2i}(f^* hr^3\partial_r f - c.c) \tag{39}$$

and thus the absorption probability for the fixed scalar ϕ_a is

$$P^{\phi_a}_{abs} = \frac{F_{horizon}}{F_\infty} = 2\pi r_1 r_5 \sqrt{r_0^2 + r_K^2}\,\omega^3 \frac{r_0^4}{4|E|^2 r_a^4}, \tag{40}$$

and the absorption cross-section for ϕ_a is then given by

$$\sigma^{\phi_a}_{abs} = \frac{9\pi^3 r_1^6 r_5^6}{64(r_1^2 + r_5^2 + \sqrt{r_1^4 + r_5^4 - r_1^2 r_5^2})^2}\omega(\omega^2 + 16\pi^2 T_L^2)(\omega^2 + 16\pi^2 T_R^2)\frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{T_L}} - 1)(e^{\frac{\omega}{T_R}} - 1)}, \tag{41}$$

which agrees with the result of [20] when $r_1 \equiv r_5$. We have restricted to the dilute gas region $r_0^2, r_K^2 \ll r_1^2, r_5^2$ and accordingly dropped terms of order r_0^2/r_1^2 and smaller. The corresponding result for the fixed scalar ϕ_b is

$$\sigma^{\phi_b}_{abs} = \frac{9\pi^3 r_1^6 r_5^6}{64(r_1^2 + r_5^2 - \sqrt{r_1^4 + r_5^4 - r_1^2 r_5^2})^2}\omega(\omega^2 + 16\pi^2 T_L^2)(\omega^2 + 16\pi^2 T_R^2)\frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{T_L}} - 1)(e^{\frac{\omega}{T_R}} - 1)}, \tag{42}$$

where we have again retained only leading order terms. Note that when $r_1 = r_5$ the absorption cross-section for λ is a factor of nine greater than that for ν.

We can use the solutions for the fixed scalars to find the gravitational perturbations δa and δb. Substitution into the two remaining Einstein equations then provides a consistency check on our results. The complexity of the graviton equations of motion implies that this check is non-trivial to perform, but it may be verified that the solutions obtained for the gravitational perturbations are indeed consistent.

III. D-BRANE ANALYSIS

The string theory prediction of the absorption cross-section for the fixed scalar ν was calculated in [19] where it was shown that the semiclassical and string cross-sections agree when $r_1 = r_5$. Now to compare the semi-classical and string emission rates we should calculate the string predictions for the cross-sections for the scalars $\phi_{a,b}$. In terms of these scalars, the action (1) takes the form

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}[R - 4(\partial\phi_a)^2 - 4(\partial\phi_b)^2 - ..]. \tag{43}$$

From [19], to study the leading order couplings of the fixed scalars it is sufficient to retain the following terms in the effective string action

$$\begin{aligned} I = \int d^2\sigma\{\frac{1}{2}(\partial_+ X\partial_- X) + \frac{1}{4T_{eff}}\nu(\partial_+ X)^2(\partial_- X)^2 \\ -\frac{1}{8T_{eff}}\lambda[\partial_+ X\partial_- X((\partial_+ X)^2 + (\partial_- X)^2) + (\partial_+ X)^2(\partial_- X)^2], \} \end{aligned} \tag{44}$$

where we have absorbed $\sqrt{T_{eff}}$ into the fields to make them properly normalised. In terms of the fields $\phi_{a,b}$, the relevant terms in the action are

$$\begin{aligned} I = \frac{1}{8T_{eff}}\int d^2\sigma\{[\phi_a(2\sin\psi - \sqrt{3}\cos\psi) + \phi_b(2\cos\psi + \sqrt{3}\sin\psi)](\partial_+ X)^2(\partial_- X)^2 \\ -\sqrt{3}[\cos\psi\phi_a + \sin\psi\phi_b]\partial_+ X\partial_- X((\partial_+ X)^2 + (\partial_- X)^2)\}. \end{aligned} \tag{45}$$

Let us consider the cross-section for the field ϕ_a. The effects of the interaction term involving the operator of dimension $(2,2)$ were considered in [19] and we can hence write down the contribution to the absorption cross-section from this interaction as

$$\sigma_{abs}^{\phi_a(1)} = \frac{\pi r_1^2 r_5^2}{1024 T_{eff}^2}(2\sin\psi - \sqrt{3}\cos\psi)^2 \; \omega(\omega^2 + 16\pi^2 T_L^2)(\omega^2 + 16\pi^2 T_R^2) \times \frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\omega}{T_L}} - 1)(e^{\frac{\omega}{T_R}} - 1)}, \tag{46}$$

We then have to consider the contributions to the cross-section from the other two interaction terms, operators of dimensions $(3,1)$ and $(1,3)$ respectively. Suppose we consider first the interaction of the fixed scalar with one right and three left movers. If p_1, p_2 and p_3 are the left moving energies and q_1 is the right moving one, the matrix element among properly normalised states is

$$\sqrt{\frac{3}{2}}\cos\psi \frac{\kappa_5}{T_{eff}} \sqrt{\frac{p_1 p_2 p_3 q_1}{\omega}}. \tag{47}$$

To compute the rate for the process $\phi_a \to L + L + L + R$ we have to square the normalised matrix element and integrate it over the possible energies of the final state particles. Because of the presence of the thermal sea of right and left movers we must insert Bose enhancement factors; for example, each left mover in the final state picks up a factor of $-\rho_L(-p_i)$ where

$$\rho_L(p_i) = \frac{1}{e^{\frac{p_i}{T_L}} - 1} \tag{48}$$

is the Bose Einstein distribution. Similarly if there is a left mover in the initial state, it picks up an enhancement factor $\rho_L(p_i)$. Conservation of energy and of momentum parallel to the effective string then introduces the factor

$$(2\pi)^2 \delta(p_1 + p_2 + p_3 + q_1 - \omega)\delta(p_1 + p_2 + p_3 - q_1) = \frac{1}{2}(2\pi)^2 \delta(p_1 + p_2 + p_3 - \frac{\omega}{2})\delta(q_1 - \frac{\omega}{2}). \tag{49}$$

To get the full rate, we should calculate first the rate for $\phi_a \to L + L + L + R$, and then include the rates for other absorption processes such as $\phi_a + L \to L + L + R$. However it is sufficient for our purposes to obtain the functional dependence of absorption rate which following the methods of [19] is given by

$$\Gamma \sim \frac{\kappa_5^2 L_{eff} \cos^2\psi}{T_{eff}^2 \omega} \int_{-\infty}^{\infty} dp_1 dp_2 dp_3 \delta(p_1 + p_2 + p_3 - \frac{\omega}{2}) \prod_{i=1,3} \frac{p_i}{1 - e^{-\frac{p_i}{T_L}}} \times \frac{\omega}{2(1 - e^{-\frac{\omega}{2T_R}})}, \tag{50}$$

where L_{eff} is the length of the effective string given by

$$\kappa_5^2 L_{eff} = 4\pi^2 r_1^2 r_5^2. \tag{51}$$

The prefactors will be determined by the number of species of left and right movers and symmetry factors. Integrating over the left movers we find that the functional dependence of the emission rate is

$$\Gamma \sim \frac{\kappa_5^2 L_{eff} \cos^2\psi}{T_{eff}^2} \frac{\omega}{(1-e^{-\frac{\omega}{2T_L}})(1-e^{-\frac{\omega}{2T_R}})} (\omega^2 + 16\pi^2 T_L^2)(\omega^2 + 32\pi^2 T_L^2), \tag{52}$$

and we obtain the same functional dependence for processes with one left and three right movers with T_L and T_R exchanged. Now the absorption cross-section is given by

$$\sigma_{abs} = \Gamma(\omega) + \Gamma(-\omega), \tag{53}$$

and thus the contribution to the ϕ_a cross-section from the $(1,3)$ and $(3,1)$ operators takes the form

$$\begin{aligned} \sigma_{abs}^{\phi_a(2)} \sim \frac{\kappa_5^2 L_{eff} \cos^2\psi}{T_{eff}^2} & \frac{\omega(e^{\frac{\omega}{T_H}}-1)}{(e^{\frac{\omega}{2T_L}}-1)(e^{\frac{\omega}{2T_R}}-1)} \times \\ & \{(\omega^2 + 16\pi^2 T_L^2)(\omega^2 + 32\pi^2 T_L^2) + (\omega^2 + 16\pi^2 T_R^2)(\omega^2 + 32\pi^2 T_R^2)\}. \end{aligned} \tag{54}$$

The cross-section for the other scalar takes a similar form with appropriate normalisation factors. Thus whatever the value of r_1 and r_5 there is no agreement between the semi-classical and string cross-sections for the fixed scalars. Even for $r_1 = r_5$, when the cross-sections for the fixed scalar $\phi_a = \nu$ are in agreement, the functional dependences of semi-classical and string cross-sections for the other fixed scalar differ.

For general charges, if we consider low energy absorption by a near extremal black hole, the temperature of the left-movers is much greater than that of the right-movers, and the dominant term in the string cross-section for ϕ_a is

$$\sigma_{abs}^{\phi_a} \rightarrow \frac{r_1^2 r_5^2 \cos^2\psi}{T_{eff}^2} T_L^5, \tag{55}$$

whereas the semi-classical cross-section behaves as

$$\sigma_{abs}^{\phi_a} \rightarrow \frac{r_1^6 r_5^6}{(r_1^2 + r_5^2 + \sqrt{r_1^4 + r_5^4 - r_1^2 r_5^2})^2} \omega^2 T_L^3, \tag{56}$$

and thus vanishes as the frequency goes to zero.

IV. COMPARISON OF GREYBODY FACTORS

We have found that for general charges the functional dependences of the string and semi-classical absorption cross-sections for the fixed scalars differ and that there is a mixing between the two scalars in the semi-classical equations which is not explained by the effective string model. It is interesting to notice that the functional dependences of both semi-classical cross-sections are determined by the behaviour in the near horizon region. At very small radius, one can see from the equations for the fixed scalars that both λ and ν satisfy hypergeometric equations. Mixing between gravitational and fixed scalar perturbations in the intermediate regions determines the normalisations of the cross-sections.

The results for the semi-classical absorption cross-sections of the fixed scalars imply that the cross-sections for both have the same functional dependence on the energy and the left

and right moving temperatures. Since the fixed scalar ν couples only to an operator of dimension $(2,2)$ in the effective string action, this suggests that the fixed scalar λ couples in a similar way. However, the expression for the string action includes couplings to operators of dimensions $(3,1)$ and $(1,3)$ and thus the semiclassical and string cross-sections disagree.

There are several possible explanations for this discrepancy. If we assume that the semi-classical calculation is correct, agreement might be restored by the presence of additional couplings within the string effective action. Indeed, since fermions had to be included in [19] in order for the normalisations of cross-sections to agree when $r_1 = r_5$, it seems feasible that further modifications of the effective action may be required. One can calculate the form of the effective string action needed to give agreement between string and semi-classical calculations as

$$I = \int d^2\sigma [\frac{3}{4T_{eff}} \phi_a (\frac{r_1^2}{(r_1^2 + r_5^2 + \sqrt{r_1^4 + r_5^4 - r_1^2 r_5^2})}) (\partial_+ X)^2 (\partial_- X)^2 \tag{57}$$

$$+ \frac{3}{4T_{eff}} \phi_b (\frac{r_1^2}{(r_1^2 + r_5^2 - \sqrt{r_1^4 + r_5^4 - r_1^2 r_5^2})}) (\partial_+ X)^2 (\partial_- X)^2], \tag{58}$$

where we have assumed $T_{eff} = 1/2\pi r_5^2$. Thus the normalisation of the interaction terms in the effective string action would have to depend on the one and five brane charges and such a dependence is difficult to explain. The importance of resolving this disagreement need not be stressed given its relationship both to the information loss question and to further understanding of the effective string model.

Note: Whilst this work was being completed, reference [21], which has considerable overlap with this paper, appeared.

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